Review session problems - 2

Math 120 Section 4

- Limits and continuity
- Partial derivatives, chain rule, tangent planes
- Maxima and minima
- Line integrals, and the FTLI
- Green's theorem (special case of Stokes')

1. Compute each limit, or show it does not exist:
   (a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy^3}{(x^2+y^2)^2}$ (b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy^2}{\sqrt{x^2+y^2}}$
   (c) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{3xy}{\sin(x^2+y^2)}$ (d) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^4y^2+1}}$

2. What are the level curves of the function $f(x,y) = \dfrac{x^2-y^2}{x^2+y^2}$?

3. Consider the surface defined implicitly by

$$\sin(xyz) = x + 2y + 3z$$

   Find the equation of the tangent plane and normal line at $(2,-1,0)$.

4. Find the local max and min and saddle points of $f(x,y) = 3xy - x^2y - xy^2$.

5. Find the absolute max and min of the function $f(x,y) = x/\sqrt{2} + y^2 - 1$ on the region D defined by $x^2+y^2 \le 1$, $x \ge 0, y \ge 0$.

6. Parametrize the curves of intersection of
   (a) the cylinder $x^2+y^2 = 16$ and the plane $x + z = 5$.
   (b) the parabolic cylinder $y = 1 - x^2$ and the plane $x + y + z = 1$.

7. Evaluate $\displaystyle\int_C \vec{F}\cdot d\vec{r}$ where $\vec{F} = y^2\cos z\vec{i} - x\vec{j} + \sin z\vec{k}$ and C is the curve $\vec{r}(t) = \langle \cos t, \sin t, t\rangle$ where $0 \le t \le 2\pi$.

8. Let $\vec{F}(x,y,z) = \langle \ln(1+y^2), \frac{2xy}{1+y^2}, z^2 \rangle$. Evaluate

$$\int_C \vec{F} \cdot d\vec{r}$$

where C is the curve given by the parametric equation $\vec{r}(t) = \langle \cos t, \sin t, t^{1/3} \rangle$ for $0 \leq t \leq 6\pi$.

9. Let $\vec{F}(x,y) = \langle y - \cos y, x \sin y + x \rangle$. Find $\int_C \vec{F} \cdot d\vec{r}$ where C is the curve $\vec{r}(t) = \langle \cos t, \sin t \rangle$ where $0 \leq t \leq 2\pi$.

10. Let $\vec{F}(x,y) = \langle \sqrt{x} + y^3, x^2 + \sqrt{y} \rangle$. Find $\int_C \vec{F} \cdot d\vec{r}$ where C is the curve given by the graph of $y = \sin x$ from $(0,0)$ to $(\pi, 0)$.