Review session problems - 1

Math 120 Section 4

- Surface integrals
- Triple Integrals
- Stokes' and Divergence theorems

1. (Spring '11) Find the surface area of the part of the paraboloid $z = x^2 + y^2$ that lies inside the sphere $x^2 + y^2 + z^2 = 6$.

2. Compute $\displaystyle\iint_S (x + y + z)dS$ where S is the surface $x^2 + y^2 = 2$, $0 \leq z \leq 2$.

3. Find the surface area of
$$\vec{r}(u, v) = \langle (2 + \cos v) \cos u, (2 + \cos v) \sin u, \sin v \rangle$$
where $0 \leq u, v \leq 2\pi$.

4. Let E be the solid region inside the cone $z = \sqrt{x^2 + y^2}$ and between the planes $z = 1$ and $z = 2$. Evaluate the following triple integral:
$$\iiint_E (x^2 + 2z)dV$$

5. (Practice Exam) Find the volume of the region lying outside the sphere $x^2 + y^2 + z^2 = 1$ and inside the sphere $x^2 + y^2 + (z - 1)^2 = 1$.

6. (Past final) Express
$$\int_0^{\sqrt{3}/2} \int_0^{\sqrt{3-4x^2}/2} \int_{1-\sqrt{1-x^2-y^2}}^{\sqrt{1-x^2-y^2}} (x + y + z)dzdydx$$
in spherical coordinates.

7. (Spring '11) Let $\vec{F}(x, y, z) = \langle 2x - y \sin z, y, e^{xy} \rangle$. Evaluate
$$\iint_S curl\vec{F} \cdot d\vec{S}$$
where S is the upper hemisphere $x^2 + y^2 + z^2 = 1$ with $z \geq 0$, oriented upward.

8. Let $\vec{F}(x,y,z) = \langle x^2yz, yz^2, z^3e^{xy}\rangle$. Evaluate

$$\iint_S curl\vec{F} \cdot d\vec{S}$$

where S is the part of the sphere $x^2+y^2+z^2=5$ that lies above the plane $z=1$, oriented upward.

9. (Spring '10) Let P be the parallelogram with vertices$A=(0,0,0)$, $B=(1,2,3)$, $C=(3,2,1)$ and $D=(4,4,4)$ oriented clockwise when viewed from above. Evaluate

$$\int_C -2010zdx + 2011xdy + 2012ydz$$

10. Evaluate $\iint_S \vec{F} \cdot d\vec{S}$ where S is part of the paraboloid $z=x^2+y^2, 0 \le z \le 1$, oriented in the positive z direction, and $\vec{F}(x,y,z) = \langle xy, xy, x^2\rangle$.

11. (Spring '10) Evaluate $\iint_S \vec{F} \cdot d\vec{S}$ where S is the surface bounded by the cylinder $x^2+y^2=1$ and the planes $z=-2$ and $z=0$, with outward pointing normals, and $\vec{F}(x,y,z) = \langle xy^2, yz^2, zx^2\rangle$.

12. (Spring '11) Let $\vec{F}(x,y,z) = \langle xz, xy, xyz\rangle$, and let E be the region bounded by the surfaces $y=1-x^2$, $y=x^2-1$ and the planes $z=0$ and $z=4$. Find the outward flux $\iint_{\partial E} \vec{F} \cdot d\vec{S}$ across the boundary ∂E.

13. Let $\vec{F}(x,y,z) = \langle x^3z, y^3z, e^{x^2+y^2}\rangle$, and let E be the solid region bounded by the cylinder $x^2+y^2=1$ and the planes $z=0$ and $x+z=2$, with the boundary surface ∂E oriented outward.
(a) Find the flux of $\vec{F}$ across the bottom face of ∂E, which is the disk $x^2+y^2=1$, $z=0$.
(b) Use the Divergence theorem to find the total flux

$$\iint_S \vec{F} \cdot d\vec{S}$$

14. (Spring '10) Evaluate $\iint_S \vec{F} \cdot d\vec{S}$ where the surface S consists of the disk $x^2+y^2 \le 4$, $z=-7$ with downward pointing normal, and cylinder $x^2+y^2=4$, $-7 \le z \le 0$ with outward pointing normal, where

$$\vec{F} = \langle ze^{y^2+1}\cos y, z+\sin x^3, x^2y^2\rangle$$