SUGGESTED EXERCISES - 1

TEICHMÜLLER THEORY (MATH 191B)

(1) Let R be a Riemann surface of genus $g = 1$. Show that the conformal automorphisms of R act transitively, that is, for any pair of points $p, q \in R$ there is a conformal homeomorphism $\phi : R \to R$ such that $\phi(p) = q$.

(2) In contrast to (1), show that if R has genus $g \geq 2$ the group of conformal automorphisms of R is finite.

(3) Show that two non-trivial elements of $PSL_2(\mathbb{R})$ commute if and only if they have the same set of fixed-points in $\overline{\mathbb{H}^2}$.

(4) Let $A \in PSL_2(\mathbb{R})$ be the hyperbolic isometry $z \mapsto \lambda^2 z$ where $1 \neq \lambda \in \mathbb{R}_+$. Show that:

$$D = \inf_{x \in \mathbb{H}^2} d(x, A \cdot x) > 0$$

and prove that in fact $D = 2|\ln \lambda|$.
(Here $d(\cdot, \cdot)$ is the hyperbolic distance.)

(5) Given an example of a Fuchsian group Γ (via generating matrices) such that the quotient $\mathbb{H}^2/\Gamma$ is a thrice-punctured sphere.

(6) Let $\Gamma = \langle \left(\begin{smallmatrix} 3 & 0 \\ 0 & 1/3 \end{smallmatrix}\right), \left(\begin{smallmatrix} 5/3 & 16/3 \\ 1/3 & 5/3 \end{smallmatrix}\right) \rangle$ be a subgroup of $PSL_2(\mathbb{R})$. Show that Γ is discrete. What is the quotient hyperbolic surface $\mathbb{H}^2/\Gamma$?

(7) Let Σ be a hyperbolic surface, and let $L > 0$. Show that there are only finitely many simple closed geodesics on Σ of length less than L.

(8) Let G be a finite group. Show that there exists a closed surface S such that there is an injective homomorphism from G into $MCG(S)$.

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