

## UM102 – Analysis and Linear Algebra II 2019 Spring Semester

[You are expected to write proofs / arguments with details of your reasoning, in solving these questions.]

### Homework Set 5 (*Quiz on Monday, February 11, in Tutorial Session*)

From the textbook *Calculus, Vol. II* by Tom M. Apostol (2nd edition):

**Section 3.11** (Exercises section), on pages 85–86:

Problems 2, 3, 5, 6 – prove the last Q6 without assuming  $C$  is square.

**Question 1.** Fix integers  $a, b, c > 0$  with  $a < b$ , and an  $a \times b$  matrix  $A$  which is **not** square. Also suppose  $M = \begin{pmatrix} A & 0 \\ 0 & B \end{pmatrix}$  is a square matrix of order  $(b + c) \times (b + c)$ .

- (1) Write down the dimensions the matrix  $B$  and the two “block off-diagonal” zero-matrices.
- (2) Compute the determinant of  $M$ .

You are allowed to use results mentioned in class – e.g. that  $\det(A) \neq 0$  if and only if  $A$  is invertible; but also results we will finish discussing on Monday, such as expansions using minors along either rows or along columns as you please; or using row operations if you wish; and also that  $\det(A) = \det(A^T)$ .