MA221 – Analysis I : Real Analysis
2017 Autumn Semester

[You are expected to write proofs / arguments with reasoning provided, in solving these questions.]

Homework Set 7 (*due by Friday, Dec 1*, in class or TA's office hours)

Question 1. Let Ω be the set of all invertible linear transformations on $\mathbb{R}^n$. Find the derivative of the map $f : \Omega \to \Omega,\ f(A) = A^{-1}$.

Question 2. Find a real valued function f defined in some neighbourhood of 0 in $\mathbb{R}^2$ such that both its partial derivatives exist at 0 yet it is not differentiable at 0.

Question 3. Let $f_n(x) = \frac{\sin nx}{x}$, $x \in \mathbb{R}$. Find the point-wise limit of f_n and check if the convergence is uniform.

Question 4. Show that no subsequence of the sequence of functions $f_n(x) = \sin nx$ converges uniformly on $[0, 2\pi]$, although, f_n is point-wise bounded.

Question 5. Suppose f_n is a equi-continuous sequence of functions defined on a compact set K and f_n converges point-wise. Prove that f_n converges uniformly on K.

Question 6. Prove the existence of a real continuous function on the real line which is nowhere differentiable.

This is Theorem 7.18 in Rudin. Go through the proof, if you like, and write the proof in your own words.