MA221 – Analysis I : Real Analysis
2017 Autumn Semester

[You are expected to write proofs / arguments with reasoning provided, in solving these questions.]

Homework Set 5 (*due by Tuesday, November 7*, in class)

Question 1. Rudin Chapter 5 Problems 2, 7, 11, 26, 27

Question 2. Suppose $f(x)$ is a polynomial function on $\mathbb{R}$, say $f(x) = c_0 + c_1x + \cdots + c_dx^d$ (of degree $d \geqslant 0$).

(a) Suppose $d > 0$ (i.e., f is non-constant) and $c_d = 1$. Prove that there exists $M > 0$ such that $f(x) > 0$ for $x > M$.

(b) With notation as in (a), prove that there exists $K > 0$ such that $f(x)$ is strictly increasing for $x > K$.

(c) Suppose $f(x) \geqslant f^{(k)}(x)$ for all $x \in \mathbb{R}$, where $f^{(k)}(x)$ denotes the kth derivative of the polynomial f. Prove that the degree d of $f(x)$ is even.

(d) If $f \geqslant f'$ on $\mathbb{R}$, then show that $f(x) \geqslant 0$ on $\mathbb{R}$. (*Hint:* Look up and use – without proof - the First Derivative Test.)

(e) If $f \geqslant f''$ on $\mathbb{R}$, then show that $f(x) \geqslant 0$ on $\mathbb{R}$. (*Hint:* Look up and use – without proof - the Second Derivative Test.)