MA221 – Analysis I : Real Analysis
2017 Autumn Semester

[You are expected to write proofs / arguments with reasoning provided, in solving these questions.]

Homework Set 2 (*due by Friday, September 8*, in TA's office hours or in class earlier in the week)

Question 1. Prove that every compact metric space is bounded. (We already showed in class that every compact set is closed; thus one implication of the Heine–Borel theorem holds in every metric space.)

Question 2. Define a closed ball in a metric space (X, d) to be $\overline{N}_r(q) := \{p \in X : d(p, q) \leqslant r\}$. Prove that a closed ball is a closed set.

Question 3. Suppose $E \subset (X, d)$ is bounded, i.e. there exists $q \in X$ and $r > 0$ such that $E \subset N_r(q)$. Prove that for every (other) point $q' \in X$, there exists $r' > 0$ such that $E \subset N_{r'}(q')$.

Question 4. Is the open interval (0,1) a disjoint countable union of closed subsets (or sub-intervals, you can decide whichever is easier to answer).

Question 5. Rudin Chapter 2 Problem 8.

Question 6. Rudin Chapter 2 Problem 10.

Question 7. Rudin Chapter 2 Problem 12.

Question 8. Rudin Chapter 2 Problem 22.

Question 9. Rudin Chapter 2 Problem 23.

Question 10. Rudin Chapter 2 Problem 25.