

Apoorva Khare: Publications

BOOKS AND SURVEY ARTICLES (PEER-REVIEWED)

- (1) **A. Khare.** *Matrix analysis and entrywise positivity preservers* (**Book**), 2022.
London Mathematical Society Lecture Note Series, Cambridge University Press, vol. 471, xxii+292 pp.; and Texts and Readings In Mathematics (TRIM Series), Hindustan Book Agency, vol. 82, xxii+339 pp.
- (2) A. Belton, D. Guillot, **A. Khare**, and M. Putinar. A panorama of positivity. I: Dimension free (survey article),
Analysis of Operators on Function Spaces (The Serguei Shimorin Memorial Volume; A. Aleman, H. Hedenmalm, D. Khavinson, M. Putinar, Eds.), pp. 117–165, Trends in Mathematics, Birkhauser, 2019.
- (3) A. Belton, D. Guillot, **A. Khare**, and M. Putinar. A panorama of positivity. II: Fixed dimension (survey article),
Complex Analysis and Spectral Theory, Proceedings of the CRM Workshop held at Laval University, QC, May 21–25, 2018 (G. Dales, D. Khavinson, J. Mashreghi, Eds.). CRM Proceedings – AMS Contemporary Mathematics 743, pp. 109–150, American Mathematical Society, 2020.
(The *unified survey* containing both parts can be found at arXiv:math.CA/1812.05482.)
- (4) **A. Khare** and A. Lachowska. *Beautiful, Simple, Exact, Crazy: Mathematics in the Real World* (**Book**, 480 pp.), Yale University Press, 2015.
- (5) **A. Khare.** Representations of complex semi-simple Lie groups and Lie algebras (invited survey article),
Connected at Infinity II: A Selection of Mathematics by Indians (R. Bhatia, C.S. Rajan, and A.I. Singh, Eds.), TRIM, Hindustan Book Agency, pp. 85–129, 2013.

PAPERS (PEER-REVIEWED)

- (6) **A. Khare.** *Probability inequalities for strongly left-invariant metric semigroups/monoids, including all Lie groups*, Indian Journal of Pure and Applied Mathematics (tribute volume in memory of K.R. Parthasarathy; 14 pp.), in press.
- (7) P.N. Choudhury and **A. Khare.** *The blowup-polynomial of a metric space: connections to stable polynomials, graphs and their distance spectra*, Canadian Journal of Mathematics (42 pp.), in press.
- (8) P.N. Choudhury and **A. Khare.** *Distance matrices of a tree: two more invariants, and in a unified framework*, European Journal of Combinatorics, 115: art.# 103787, 30 pp., 2024.
- (9) A. Belton, D. Guillot, **A. Khare**, and M. Putinar. *Totally positive kernels, Pólya frequency functions, and their transforms*, Journal d’Analyse Mathématique, 150(1):83–158, 2023.
- (10) **A. Khare** and S. Sahi. *From Cauchy’s determinant formula to bosonic and fermionic immanant identities*, European Journal of Combinatorics, 110: art.# 103683, 16 pp., 2023; erratum, Eur. J. Combin. 112: art.# 103743, 2 pp., 2023.
- (11) **A. Khare** and B. Rajaratnam. *The Khinchin–Kahane and Lévy inequalities for abelian metric groups, and transfer from normed (abelian semi)groups to Banach spaces*, Journal of Mathematical Analysis and Applications, 528(2): art.# 127545, 18 pp., 2023.
- (12) **A. Khare** and A. Tikaradze. *Recovering affine-linearity of functions from their restrictions to affine lines*, Journal of Algebraic Combinatorics, 58(3):761–773, 2023.

PAPERS (PEER-REVIEWED) (CONT.)

- (13) A. Belton, D. Guillot, **A. Khare**, and M. Putinar. *Moment-sequence transforms*, Journal of the European Mathematical Society, 24(9):3109–3160, 2022.
- (14) A. Belton, D. Guillot, **A. Khare**, and M. Putinar. *Matrix compression along isogenic blocks*, Acta Scientiarum Mathematicarum (Szeged), 88(1–2) – 100th anniversary special volume, 417–448, 2022.
- (15) **A. Khare**. *Smooth entrywise positivity preservers, a Horn–Loewner master theorem, and symmetric function identities*, Transactions of the American Mathematical Society, 375(3):2217–2236, 2022.
- (16) G. Dhillon and **A. Khare**. *The weights of simple modules in Category \mathcal{O} for Kac–Moody algebras*, Journal of Algebra, 603:164–200, 2022.
- (17) A. Belton, D. Guillot, **A. Khare**, and M. Putinar. *Hirschman–Widder densities*, Applied and Computational Harmonic Analysis, 60:396–425, 2022.
- (18) A. Belton, D. Guillot, **A. Khare**, and M. Putinar. *Preservers of totally positive kernels and Pólya frequency functions*, Mathematics Research Reports, 3:35–56, 2022. [Joint announcement of 3 papers: arXiv/2006.16213 (*J. d’Analyse Math.*, 2023), arXiv/2008.05121 (preprint, 2020), arXiv/2101.02129 (*Appl. Comp. Harm. Anal.*, 2022).]
- (19) **A. Khare** and A. Tikaradze. *Covering modules by proper submodules*, Communications in Algebra, 50(2):498–507, 2022.
- (20) L. Bouthat, **A. Khare**, J. Mashreghi, and F. Morneau-Guérin. *The p -norm of circulant matrices*, Linear and Multilinear Algebra, 70(21):7176–7188, 2022.
- (21) **A. Khare** and T. Tao. *On the sign patterns of entrywise positivity preservers in fixed dimension*, American Journal of Mathematics, 143(6):1863–1929, 2021.
- (22) **A. Khare**. *Sharp nonzero lower bounds for the Schur product theorem*, Proceedings of the American Mathematical Society, 149(12):5049–5063, 2021.
- (23) P.N. Choudhury, M.R. Kannan, and **A. Khare**. *Sign non-reversal property for totally non-negative and totally positive matrices, and testing total positivity of their interval hull*, Bulletin of the London Mathematical Society, 53(4):981–990, 2021.
- (24) **A. Khare** and V.P. Pingali. *On an asymptotic characterisation of Griffiths semipositivity*, Bulletin des Sciences Mathématiques, 167: art.# 102956, 7 pp., 2021.
- (25) **A. Khare** and B. Rajaratnam. *Differential calculus on the space of countable labelled graphs*, Advances in Operator Theory, 6(1): art.# 12, 28 pp., 2021.
- (26) **A. Khare** and B. Rajaratnam. *Probability inequalities and tail estimates for metric semi-groups*, Advances in Operator Theory, 5(3):779–795, 2020 (Special issue in honor of Rajendra Bhatia).
- (27) C. Benedetti, R.S. González D’León, C.R.H. Hanusa, P.E. Harris, **A. Khare**, A.H. Morales, and M. Yip. *A combinatorial model for computing volumes of flow polytopes*, Transactions of the American Mathematical Society, 372(5):3369–3404, 2019.
- (28) A. Belton, D. Guillot, **A. Khare**, and M. Putinar. *Simultaneous kernels of matrix Hadamard powers*, Linear Algebra and its Applications 576:142–157, 2019.
- (29) **A. Khare** and A. Tikaradze. *A Carlitz–von Staudt type theorem for finite rings*, Linear Algebra and its Applications 568:106–126, 2019.

PAPERS (PEER-REVIEWED) (CONT.)

- (30) (As D.H.J. Polymath:) T. Fritz, S. Gadgil, **A. Khare**, P. Nielsen, L. Silberman, and T. Tao. *Polymath14: Homogeneous length functions on groups*, Algebra & Number Theory 12(7):1773–1786, 2018.
- (31) **A. Khare**. *Generalized nil-Coxeter algebras over discrete complex reflection groups*, Transactions of the American Mathematical Society 370(4):2971–2999, 2018.
- (32) G. Dhillon and **A. Khare**. *Faces of highest weight modules and the universal Weyl polyhedron*, Advances in Mathematics 319:111–152, 2017.
- (33) **A. Khare** and B. Rajaratnam. *The Hoffmann-Jørgensen inequality in metric semigroups*, Annals of Probability 45(6A):4101–4111, 2017.
- (34) D. Guillot, **A. Khare**, and B. Rajaratnam. *Preserving positivity for rank-constrained matrices*, Transactions of the American Mathematical Society 369(9):6105–6145, 2017.
- (35) **A. Khare**. *Standard parabolic subsets of highest weight modules*, Transactions of the American Mathematical Society 369(4):2363–2394, 2017; erratum, Trans. AMS 369(4):3015, 2017.
- (36) **A. Khare**. *Generalized nil-Coxeter algebras, cocommutative algebras, and the PBW property*, AMS Contemporary Mathematics 688:139–168, 2017 (Special issue in honor of Donald S. Passman).
- (37) A. Belton, D. Guillot, **A. Khare**, and M. Putinar. *Matrix positivity preservers in fixed dimension. I*, Advances in Mathematics 298:325–368, 2016.
- ———, Announcement of paper, in: *Matrix positivity preservers in fixed dimension*, Comptes Rendus Mathématique de l’Académie des Sciences 354(2):143–148, 2016.
- (38) D. Guillot, **A. Khare**, and B. Rajaratnam. *Critical exponents of graphs*, Journal of Combinatorial Theory, Series A 139:30–58, 2016.
- (39) **A. Khare**. *Faces and maximizer subsets of highest weight modules*, Journal of Algebra 455:32–76, 2016.
- (40) **A. Khare** and A. Tikaradze. *On Category \mathcal{O} over triangular generalized Weyl algebras*, Journal of Algebra 449:687–729, 2016.
- (41) D. Guillot, **A. Khare**, and B. Rajaratnam. *Preserving positivity for matrices with sparsity constraints*, Transactions of the American Mathematical Society 368(12):8929–8953, 2016.
- (42) P. Diao, D. Guillot, **A. Khare**, and B. Rajaratnam. *Differential calculus on graphon space*, Journal of Combinatorial Theory, Series A 133:183–227, 2015.
- (43) D. Guillot, **A. Khare**, and B. Rajaratnam. *Complete characterization of Hadamard powers preserving Loewner positivity, monotonicity, and convexity*, Journal of Mathematical Analysis and Applications 425(1):489–507, 2015.
- (44) D. Guillot, **A. Khare**, and B. Rajaratnam. *The critical exponent conjecture for powers of doubly nonnegative matrices*, Linear Algebra and its Applications 439(8):2422–2427, 2013.
- (45) **A. Khare** and T.B. Ridenour. *Faces of weight polytopes and a generalization of a theorem of Vinberg*, Algebras and Representation Theory 15(3):593–611, 2012.
- (46) V. Chari, **A. Khare**, and T.B. Ridenour. *Faces of polytopes and Koszul algebras*, Journal of Pure and Applied Algebra 216(7):1611–1625, 2012.
- (47) **A. Khare**. *The sum of a finite group of weights of a Hopf algebra*, Axioms 1(3):259–290, 2012 (Special issue on: “Hopf Algebras, Quantum Groups and Yang-Baxter Equations”).

PAPERS (PEER-REVIEWED) (CONT.)

- (48) A. Khare, A. Saxena, and **A. Khare**. *Solutions of several coupled discrete models in terms of Lamé polynomials of arbitrary order*, *Pramana* 79(3):377–392, 2012.
- (49) **A. Khare** and A. Tikaradze. *Center and representations of infinitesimal Hecke algebras of \mathfrak{sl}_2* , *Communications in Algebra* 38(2):405–439, 2010.
- (50) **A. Khare**. *Functoriality of the BGG Category \mathcal{O}* , *Communications in Algebra* 37(12):4431–4475, 2009.
- (51) **A. Khare**. *Vector spaces as unions of proper subspaces*, *Linear Algebra and its Applications* 431(9):1681–1686, 2009.
- (52) W.L. Gan and **A. Khare**. *Quantized symplectic oscillator algebras of rank one*, *Journal of Algebra* 310(2):671–707, 2007.
- (53) **A. Khare**. *Divisibility tests and recurring decimals in Euclidean domains* (final section with P. Moree), *JP Journal of Algebra, Number Theory, and Appl. (JPANTA)* 7(1):1–32, 2007.
- (54) **A. Khare**. *Category \mathcal{O} over the symplectic oscillator algebra*, Ph.D. Thesis, University of Chicago, 2006.
- (55) **A. Khare**. *Category \mathcal{O} over a deformation of the symplectic oscillator algebra*, *Journal of Pure and Applied Algebra* 195(2):131–166, 2005; erratum, *JPAA* 199:319–320, 2005.
- (56) **A. Khare**. *Divisibility tests*, *Furman University Electronic Journal of Undergraduate Mathematics (FUEJUM)* 3:1–5, 1997.

CONFERENCE PROCEEDINGS – FPSAC

(INTERNATIONAL CONFERENCE ON “FORMAL POWER SERIES AND ALGEBRAIC COMBINATORICS”)

- (57) P.N. Choudhury and **A. Khare**. *Blowup polynomials and delta-matroids of graphs* (extended abstract of the paper “*The blowup-polynomial of a metric space: connections to stable polynomials, graphs and their distance spectra*”), 34th FPSAC, 2022; Séminaire Lotharingien de Combinatoire 86B, Article #42 (12 pages), 2022.
- (58) **A. Khare** and T. Tao. *Schur polynomials, entrywise positivity preservers, and weak majorization* (extended abstract of the paper “*On the sign patterns of entrywise positivity preservers in fixed dimension*”), 30th FPSAC, 2018; Séminaire Lotharingien de Combinatoire 80B, Article #14 (12 pages), 2018.
- (59) **A. Khare**. *Generalized nil-Coxeter algebras* (extended abstract of the paper “*Generalized nil-Coxeter algebras over discrete complex reflection groups*”), 30th FPSAC, 2018; Séminaire Lotharingien de Combinatoire 80B, Article #29 (12 pages), 2018.
- (60) C. Benedetti, R.S. González D’León, C.R.H. Hanusa, P.E. Harris, **A. Khare**, A.H. Morales, and M. Yip. *The volume of the caracol polytope* (extended abstract of the paper “*A combinatorial model for computing volumes of flow polytopes*”), 30th FPSAC, 2018; Séminaire Lotharingien de Combinatoire 80B, Article #87 (12 pages), 2018.
- (61) G. Dhillon and **A. Khare**. *The Weyl–Kac weight formula* (extended abstract of the paper “*Characters of highest weight modules and integrability*”), Talk at the 29th FPSAC, 2017; Séminaire Lotharingien de Combinatoire 78B, Article #77 (12 pages), 2017.
- (62) D. Guillot, **A. Khare**, and B. Rajaratnam. *The critical exponent: a novel graph invariant* (extended abstract of the paper “*Critical exponents of graphs*”), 29th FPSAC, 2017; Séminaire Lotharingien de Combinatoire 78B, Article #62 (12 pages), 2017.

- (63) A. Belton, D. Guillot, **A. Khare**, and M. Putinar. *Schur polynomials and matrix positivity preservers* (extended abstract of the paper “*Matrix positivity preservers in fixed dimension. I*”), 28th FPSAC, 2016; DMTCS Proceedings BC, pages 155–166, 2016.

ADDITIONAL RESEARCH OUTPUT

- (1) **A. Khare** and T. Tao. *Schur polynomials, entrywise positivity preservers, and weak majorization*, Poster presented at the 30th International Conference on Formal Power Series and Algebraic Combinatorics (FPSAC '18), 2018.
- (2) **A. Khare**. *Generalized nil-Coxeter algebras*, Poster presented at the 30th International Conference on Formal Power Series and Algebraic Combinatorics (FPSAC '18), 2018.
- (3) C. Benedetti, R.S. González D'León, C.R.H. Hanusa, P.E. Harris, **A. Khare**, A.H. Morales, and M. Yip. *The volume of the caracol polytope*, Poster at the 30th International Conference on Formal Power Series and Algebraic Combinatorics (FPSAC '18), 2018.
- (4) A. Belton, D. Guillot, **A. Khare**, and M. Putinar. *Schur polynomials and matrix positivity preservers*, Poster presented at the 28th International Conference on Formal Power Series and Algebraic Combinatorics (FPSAC '16), 2016.
- (5) D. Guillot, **A. Khare**, and B. Rajaratnam. Concept note by organizers, Workshop on *Positivity, graphical models, and modeling of complex multivariate dependencies*, American Institute of Mathematics (AIM), 2014.
- (6) **A. Khare** and B. Rajaratnam. *Rigorous mathematical foundations for network analysis*, Poster (Mathematics) presented at the DARPA Young Faculty Award KickOff Meeting, Washington D.C., 2012.
- (7) **A. Khare** and B. Rajaratnam. *Rigorous mathematical foundations for network analysis*, Poster (Mathematics) presented at the DARPA Young Faculty Award KickOff Meeting, Washington D.C., 2011.

OTHER OUTPUT (EXPOSITORY ARTICLE; INTERVIEW; GUEST EDITOR)

- (8) **A. Khare**. *Polymath 14: Groups with norms* (extended abstract of the paper “*Polymath14: Homogeneous length functions on groups*”), 88th Annual Conference of the Indian Math. Society, 2022; Math. Student 92(1-2), pages 127–136, 2023.
- (9) *Interview of Rajendra Bhatia* by **A. Khare**, IMAGE (bulletin of the International Linear Algebra Society), Issue 70, pages 3–10, 2023.
- (10) R. Ganapathy, **A. Khare**, and R. Venkatesh (Guest Editors), Journal of the Indian Institute of Science, Special volume on *Number theory and representation theory*: Volume 102 issue 3, 2022.

PREPRINTS

- (1) S.S. Damase and **A. Khare**. *A remark on characterizing inner product spaces via strong three-point homogeneity* (2023), arXiv:math.FA/2312.08106.
- (2) A. Belton, D. Guillot, **A. Khare**, and M. Putinar. *Negativity-preserving transforms of tuples of symmetric matrices* (2023), arXiv:math.CA/2310.18041.
- (3) A. Belton, D. Guillot, **A. Khare**, and M. Putinar. *Matrix positivity preservers in fixed dimension. II: positive definiteness and strict monotonicity of Schur function ratios* (2023), arXiv:math.CA/2310.18020.

PREPRINTS (CONT.)

- (4) P.N. Choudhury and **A. Khare**. *The additive-multiplicative distance matrix of a graph, and a novel third invariant* (2023), arXiv:math.CO/2309.08691.
- (5) **A. Khare** and G. Krishna Teja. *A weight-formula for all highest weight modules, and a higher order parabolic category \mathcal{O}* (2022), arXiv:math.RT/2203.05515.
- (6) **A. Khare**. *Matrix analysis, Pólya frequency functions, and preservers of (total) positivity*, Lecture notes (320+ pp.), available on author's website and under revision.
- (7) S. Bhattacharya and **A. Khare**. *The lattice of nil-Hecke algebras over real and complex reflection groups* (2021), arXiv:math.RA/2111.14800.
- (8) P.N. Choudhury and **A. Khare**. *Multivariate blowup-polynomials of graphs* (2021), arXiv:math.CO/2106.03751.
- (9) **A. Khare**. *Multiply positive functions, critical exponent phenomena, and the Jain–Karlin–Schoenberg kernel* (2020), arXiv:math.FA/2008.05121.
- (10) **A. Khare**. *The non-compact normed space of norms on a finite-dimensional Banach space* (2018), arXiv:math.CA/1810.06188.
- (11) P. Diao, D. Guillot, **A. Khare**, and B. Rajaratnam. *Model-free consistency of graph partitioning* (2016), arXiv:math.CO/1608.03860.
- (12) **A. Khare** and B. Rajaratnam. *Integration and measures on the space of countable labelled graphs*, preprint (under revision), arXiv:math.CA/1506.01439.
- (13) **A. Khare**. *Axiomatic framework for the BGG Category \mathcal{O}* , preprint (under revision), arXiv:math.RT/1502.06706.
- (14) D. Guillot, **A. Khare**, and B. Rajaratnam. *On fractional Hadamard powers of positive block matrices*, preprint (under revision), arXiv:math.FA/1404.6839.