Quiz 2

August 16, 2018

Question: Suppose that a sequence $\{a_n\}$ has limit L, and $c \in \mathbb{R}$. Then show from first principles that the sequence $\{a_n + c\}$ converges by quessing it's limit and proving that the limit satisfies the desired properties.

Solution:
Claim: $\{a_n + c\}$ converges to $L + c$ for any $c \in \mathbb{R}$.

Let $\epsilon > 0$ is given. We want a $n_0 \in \mathbb{N}$ such that

$$|(a_n + c) - (L + c)| < \epsilon \quad \forall n \geq n_0.$$

Since $\{a_n\}$ converges to L, therefore $\exists\ m_0 \in \mathbb{N}$ such that

$$|a_n - L| < \epsilon \quad \forall n \geq m_0.$$

But

$$|(a_n + c) - (L + c)| = |a_n - L| \quad \forall n \in \mathbb{N},$$

hence

$$|(a_n + c) - (L + c)| = |a_n - L| < \epsilon \quad \forall n \geq m_0.$$

Which proves that $\{a_n + c\}$ converges to $L + c$.