Quiz 1

August 9, 2018

Question: Suppose X is a set and A, $B \subset X$. Then prove from first principles that

$$X \setminus (A \cap B) = (X \setminus A) \cup (X \setminus B)$$

Solution: Take C, $D \subset X$ and $C \subset D$. We show that $(X \setminus D) \subset (X \setminus C)$. Let $x \in (X \setminus D)$. So, $x \in X$ and $x \notin D$. But from $C \subset D$, we have $x \in C \Rightarrow x \in D$. Hence, $x \notin D \Rightarrow x \notin C$. Thus, $x \in X$ and $x \notin D \Rightarrow x \in X$ and $x \notin C$. Hence $(X \setminus D) \subset (X \setminus C)$.
Using the result above, as $(A \cap B) \subset A \subset X$, we have $(X \setminus A) \subset X \setminus (A \cap B)$.
Again using the result above, as $(A \cap B) \subset B \subset X$, we have $(X \setminus B) \subset X \setminus (A \cap B)$.
Hence, $(X \setminus A) \cup (X \setminus B) \subset X \setminus (A \cap B)$.

Let $x \in X \setminus (A \cap B)$. So, $x \in X$, and $x \notin (A \cap B)$.
$x \notin (A \cap B) \Rightarrow x \notin A$ or $x \notin B$. For $x \in X$ and $x \notin A$, we have $x \in (X \setminus A)$. And for $x \in X$ and $x \notin B$, we have $x \in (X \setminus B)$. Hence, $x \in X \setminus (A \cap B) \Rightarrow x \in (X \setminus A)$ or $x \in (X \setminus B)$. Thus, we have $X \setminus (A \cap B) \subset (X \setminus A) \cup (X \setminus B)$.
Hence, from the first principles we can conclude

$$X \setminus (A \cap B) = (X \setminus A) \cup (X \setminus B)$$