

SOLUTION OF QUIZ 11

We need to show that $T(p(x) + q(x)) = T(p(x)) + T(q(x))$ and $T(c.p(x)) = c.T(p(x))$ $\forall p, q \in V$ and $\forall c \in \mathbb{R}$. Let $h(x) = p(x) + q(x)$. Then, $h(x+1) = p(x+1) + q(x+1)$. Thus, $T(p(x) + q(x)) = T(p(x)) + T(q(x)) \forall p, q \in V$. Now, let $g(x) = c.p(x)$. So, $g(x+1) = c.p(x+1)$. Thus, $T(c.p(x)) = c.T(p(x)) \forall c \in \mathbb{R}$. Hence, $T \in \mathcal{L}(V)$.

$$T(1) = 1.$$

$$T(1+x) = x+2 = (1+x) + 1.$$

$$T(1+x^2) = 1 + (x+1)^2 = x^2 + 2x + 2 = (1+x^2) + 2(1+x) + (-1)(1).$$

$$T(1+x^3) = 1 + (x+1)^3 = x^3 + 3x^2 + 3x + 2 = (1+x^3) + 3(1+x^2) + 3(1+x) + (-5)(1).$$