Solution Of Quiz 10

Solution : First consider the subset $W_1 = \{f : [0,1] \to \mathbb{R} : f(x) = f(1-x) \ \forall x \in [0,1]\}$. Clearly W_1 is non empty as the constant 0 function is in W_1. W_1 is a subset of the vector space V. We know that a non empty subset of a vector space is a subspace of that vector space iff it is closed under addition and scalar multiplication. Let $f, g \in W_1$, i.e. $f : [0,1] \to \mathbb{R}$, $g : [0,1] \to \mathbb{R}$ with $f(x) = f(1-x)$ for all $x \in [0,1]$, $g(x) = g(1-x)$ for all $x \in [0,1]$. Let $c_1, c_2 \in \mathbb{R}$. See that $(c_1f+c_2g)(x) = c_1f(x)+c_2g(x) = c_1f(1-x)+c_2g(1-x) = (c_1f + c_2g)(1-x)$ for all $x \in [0,1]$. Hence $c_1f + c_2g \in W_1$, i.e. W_1 satisfies the closure property. Hence W_1 is a subspace of V.

Now consider $W_2 = \{f : [0,1] \to \mathbb{R} : f(x) = 1 + f(1-x) \ \forall x \in [0,1]\}$. Clearly see that the 0 element of V is not in W_2, hence it is not a subspace.