Homework 9

Analysis and Linear Algebra I (Autumn 2018)
Indian Institute of Science

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October 12, 2018

1. Exercises 1, 5, 9, 12, 19, 20 of Section 1.26 in the textbook.

2. Find a cubic polynomial P such that $P(0) = P(-2) = 0$, $P(1) = 15$ and $\int_{-2}^{0} P(x)\, \mathrm{d}x = 4$. Is it unique?

3. If $a < b < c < d$, and f is integrable on $[a, d]$, prove that f is integrable on $[b, c]$.

4. Let f be defined on the interval $[-a, a]$ for some $a > 0$. f is called an *even* (resp. *odd*) function if $f(-x) = f(x)$ (resp. $f(-x) = -f(x)$) for all $x \in [-a, a]$. If f is an even or odd function integrable on $[0, a]$, prove that it is also integrable on $[-a, a]$ and satisfies

   (a) $\int_{-a}^{a} f(x)\, \mathrm{d}x = 2 \int_{0}^{a} f(x)\, \mathrm{d}x$ if f is even, and

   (b) $\int_{-a}^{a} f(x)\, \mathrm{d}x = 0$ if f is odd.

5. Prove that if f is integrable on $[a, b]$ and $\epsilon > 0$, then there exists a step function s such that $\int_{a}^{b} f(x)\, \mathrm{d}x - \int_{a}^{b} s(x)\, \mathrm{d}x < \epsilon$. (**Harder variant**: can you replace the step function s by a continuous function g satisfying the same property?)

6. Exercises 1, 4, 9, 14 of Section 2.4 in the textbook.

7. Give a quick argument to show that the area enclosed by the ellipse $(x/a)^2 + (y/b)^2 = 1$ for $a, b > 0$ is measurable and calculate it using the properties of the integral.

8. Exercises 1, 3, 8 of Section 2.17 in the textbook.