# Homework 8

Analysis and Linear Algebra I (Autumn 2018)
Indian Institute of Science

Instructor: Arvind Ayyer

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1. Exercises 5, 8, 10 of Section 7.4 in the textbook.

2. Show that $r = \sqrt{15} - 3$ is an approximation to the non-zero solution of the equation $x^2 = \sin x$ by using $T_3(\sin x)$.

3. Draw the graphs of the following step functions, where $[x]$ is the greatest integer $\leq x$.

   (a) $[x] + [2x]$ in $[-1, 2]$
   (b) $[x] \cdot [2x]$ in $[-1, 2]$
   (c) $[2x] \cdot [x/2]$ in $[-1, 2]$
   (d) $[\sqrt{x}]$ in $[0, 10]$
   (e) $\sqrt{[x]}$ in $[0, 10]$

4. Evaluate the following integrals of step functions

   (a) $\displaystyle\int_{-1}^{3} [2x] \, \mathrm{d}x$
   (b) $\displaystyle\int_{0}^{n} [x] \, \mathrm{d}x$, where $n \in \mathbb{N}$
   (c) $\displaystyle\int_{a}^{b} [x] \, \mathrm{d}x + \int_{a}^{b} [-x] \, \mathrm{d}x$ for $a, b \in \mathbb{R}$
   (d) $\displaystyle\int_{0}^{n^2} [\sqrt{x}] \, \mathrm{d}x$

5. Prove Theorems 1.2 – 1.8 (see the hints in exercises 12-17 in Section 1.15)

6. Prove that if $f$ and $g$ are integrable on $[a, b]$ such that $f(x) \geq g(x)$ for all $x \in [a, b]$, then
$$\int_a^b f(x) \, \mathrm{d}x \geq \int_a^b g(x) \, \mathrm{d}x.$$

7. If $a, b \in \mathbb{R}$ and $a < b$ such that $f : [a, b] \to \mathbb{R}$ is integrable, prove that
$$\int_a^b f(x) \, \mathrm{d}x = (b - a) \int_0^1 f\left(a + (b - a)x\right) \, \mathrm{d}x.$$