Homework 7

Analysis and Linear Algebra I (Autumn 2018)
Indian Institute of Science

Instructor: Arvind Ayyer

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1. Determine $g'(x)$ in terms of $f'(x)$ when

   (a) $g(x) = f(x^2)$,
   (b) $g(x) = f(\tan x)$,
   (c) $g(x) = (f \circ f)(x)$,
   (d) $g(x) = (f \circ f)(x^2)$.

2. Exercises 20, 23, 26, 29, 30 and 33 of Section 4.12 in the textbook.

3. Verify the mean value theorem for $f(x) = 2x^2 - 7x + 10$ in the interval $[2, 5]$.

4. For a quadratic polynomial, prove that the tangent at the midpoint of the interval $[a, b]$ has the same slope as the chord joining the endpoints of the graph.

5. Let $c_0, c_1, \ldots, c_n \in \mathbb{R}$ such that $\displaystyle\sum_{k=0}^{n} \frac{c_k}{k+1} = 0$.
   Prove that the equation $c_0 + c_1 x + \cdots c_n x^n = 0$ has at least one real root.

6. Use the mean value theorem to conclude that $|\sin x - \sin y| \le |x - y|$ for all $x, y \in \mathbb{R}$.

7. Let f be a function continuous on a closed interval $[a, b]$. Assume that $f''(x)$ exists at each $x \in (a, b)$. Suppose the line segment in the plane joining $(a, f(a))$ to $(b, f(b))$ intersects the graph of f at a third point $(c, f(c))$, where $c \in (a, b)$. Show that $f''(x_0) = 0$ for some point $x_0 \in (a, b)$.

8. Suppose f is twice differentiable on $[a, b]$ and satisfies

   $$f''(x) + f'(x)g(x) - f(x) = 0$$

   for some function g. Prove that if $f(a) = f(b) = 0$, then $f(x) = 0$ for all $x \in (a, b)$.

9. Exercises 1, 8, 9, 12, 14 of Section 4.19 in the textbook.