Homework 4

Analysis and Linear Algebra I (Autumn 2018)
Indian Institute of Science

Instructor: Arvind Ayyer

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1. Determine whether the convergent series in Problem 5 of Homework 3 converge conditionally or absolutely.

2. If $\{a_n\}$ and $\{b_n\}$ are two sequences such that $a_n > 0$ and $b_n > 0$ for all $n \geq N$, and $c_n = b_n - \frac{b_{n+1}\, a_{n+1}}{a_n}$, then prove that

   (a) If $\exists\, r > 0$ such that $c_n \geq r > 0$ for all $n \geq N$, then $\sum a_n$ converges, and

   (b) If $c_n \leq 0$ for all $n \geq N$ and if $\sum 1/b_n$ diverges, then $\sum a_n$ diverges.

3. If $\sum_n a_n$ converges absolutely, then prove that $\sum_n a_n^2$ converges. Is the converse true?

4. If the left hand limit and the right hand limit of f at p are not equal, prove that the limit does not exist.

5. Compute the limits and prove that the limits are correct from first principles for the following problems.

   (a) $\lim_{x \to 1} \dfrac{1}{x}$ (b) $\lim_{x \to 2} \dfrac{1}{x^2}$

   (c) $\lim_{x \to 1} \sqrt{x}$ (d) $\lim_{x \to 0} \dfrac{x}{x}$