Homework 3

Analysis and Linear Algebra I (Autumn 2018)
Indian Institute of Science

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1. Prove that the following series converge to the values indicated

   (a) $\displaystyle\sum_n \frac{1}{4n^2-1} = \frac{1}{2}$.

   (b) $\displaystyle\sum_n \frac{2}{3^{n-1}} = 3$.

   (c) $\displaystyle\sum_n \frac{2^n + n + n^2}{2^{n+1}\, n(n+1)} = 1$.

2. Compute the sums of the following series. In each case, clearly state the values of x for which the series converges.

   (a) $\displaystyle\sum_n 4^n x^{2n}$

   (b) $\displaystyle\sum_n (x^n - x^{2n})$

3. Use comparison tests to determine whether the following series converge or diverge. You do not need to determine the value if it converges.

   (a) $\displaystyle\sum_n \frac{n}{(4n-1)(4n-3)}$

   (b) $\displaystyle\sum_n \frac{n+1}{2^n}$

   (c) $\displaystyle\sum_n \frac{1+\sqrt{n}}{(1+n)^3 - 1}$

   (d) $\displaystyle\sum_n \frac{n^2}{2^n}$

4. Determine whether the following series converge or diverge. You do not need to determine the value if it converges. You may assume (10.9)-(10.14) of the textbook.

   (a) $\displaystyle\sum_n \frac{(n!)^2}{(2n)!}$

   (b) $\displaystyle\sum_n \frac{2^n\, n!}{n^n}$

   (c) $\displaystyle\sum_n e^{-n^2}$

   (d) $\displaystyle\sum_n \frac{n^{n+1}}{(n+1/n)^n}$

5. Determine whether the following series converge or diverge. You do not need to determine the value if it converges.

(a) $\displaystyle\sum_n \frac{(-1)^{n(n-1)/2}}{2^n}$

(b) $\displaystyle\sum_n \frac{(-1)^n}{\sqrt{n}+(-1)^n}$

(c) $\displaystyle\sum_n \frac{(-1)^n\sqrt{n}}{n+100}$

(d) $\displaystyle\sum_n \frac{\sin(1/n)}{n}$