Homework 14
Analysis and Linear Algebra I (Autumn 2018)
Indian Institute of Science

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1. Exercises 2, 3, 6, 7, 12 of Section 16.16 in the textbook.

2. Exercises 2, 4, 6, 9 of Section 16.20 in the textbook.

3. Let V be the vector space of real polynomials and $D \in \mathcal{L}(V)$ be the differentiation operator. Let $T : V \to V$ be defined by $T(p(x)) = xp'(x)$.

   - Show that T is linear.
   - Determine all polynomials p such that $Tp = p$.
   - Determine all polynomials p such that $(DT - 2D)p = 0$.

4. Find all 2×2 matrices such that $A^2 = 0$.

5. If A is a square matrix such that $A^2 = A$, prove that $(A + I)^k = I + (2k1)A$.

6. Prove or disprove: if A and B are invertible, then $A + B$ is invertible.

7. Prove that $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is invertible if and only if $ad - bc \neq 0$. In that case, determine its inverse.