Homework 13

Analysis and Linear Algebra I (Autumn 2018)
Indian Institute of Science

Instructor: Arvind Ayyer

November 9, 2018

1. Exercises 1, 3, 4, 6, 10, 12 of Section 15.12 in the textbook.
2. Prove that, for all x, y in an inner product space V,
$$||x+y||^2 + ||x-y||^2 = 2\,||x||^2 + 2\,||y||^2.$$
3. Exercises 1, 4, 6, 10, 18, 20 of Section 16.4 in the textbook.
4. Exercises 11, 14, 15 of Section 16.4 in the textbook.
5. Exercises 24, 25 of Section 16.4 in the textbook.
6. Exercises 3, 6, 11, 15, 20 of Section 16.8 in the textbook.
7. Let $T\colon V \to V$ be a linear transformation and multiplication of transformations be defined by composition as usual. Show that $T^{m+n} = T^m \cdot T^n$ for all $m, n \in \mathbb{N}$. Moreover, if T is invertible, with $T(V) = V$, then prove that T^n is also invertible and that $(T^n)^{-1} = (T^{-1})^n$.
8. We say that two linear transformations $S, T \in \mathcal{L}(V)$ **commute** if $ST = TS$. Give a general formula for $(S+T)^n$, where S and T commute and $n \in \mathbb{N}$. (**More difficult**: Can you give a general formula when S and T don't commute?)
9. Let S, T be two invertible linear transformations. Prove that $(ST)^{-1} = T^{-1}S^{-1}$.
10. Let $S, T\colon \mathbb{R}^3 \to \mathbb{R}^3$ be defined as follows. If $(x,y,z) \in \mathbb{R}^3$, then $S(x,y,z) = (z,y,x)$ and $T(x,y,z) = (x, x+y, x+y+z)$. Prove that S and T are injective, and give explicit description for the action of $S^{-1}, T^{-1}, ST, TS, (ST)^{-1}$ and $(TS)^{-1}$.
11. Exercises 2, 4, 5, 13, 16 of Section 16.12 in the textbook.