

Homework 12

Analysis and Linear Algebra I (Autumn 2018)
Indian Institute of Science

Instructor: Arvind Ayyer

November 2, 2018

1. Exercises 2, 4, 6, 9, 11 of Section 15.5 in the textbook.
2. Exercises 13, 14, 16 of Section 15.5 in the textbook.
3. Exercises 18, 21 of Section 15.5 in the textbook.
4. Exercises 23, 24, 25, 27 of Section 15.5 in the textbook.
5. Let $F = \mathbb{R}$. Define the set $V = \mathbb{R}_+ = \{x \in \mathbb{R}, x > 0\}$ and the operations \oplus, \otimes as follows:

$$x \oplus y = x \cdot y \text{ for } x, y \in V \text{ and } c \otimes x = x^c \text{ for } c \in F, x \in V.$$

Prove that V is a vector space over F under these operations. What elements play the role of 0 and 1 in V ?

6. Exercises 2, 5, 7, 8, 11, 16 of Section 15.9 in the textbook.
7. Find all real t such that the two vectors $(1+t, 1-t)$ and $(1-t, 1+t)$ in \mathbb{R}^2 are independent.
8. Prove that a subset S of a vector space V is a subspace if and only if $L(S) = S$.
9. Find the dimension of the subspace $L(S)$ for each of the following sets S in the given vector space V .
 - (a) V : polynomials of degree at most 5, $S = \{1, x^2, x^5\}$.
 - (b) V : polynomials of degree at most 5, $S = \{1 + x, (1 + x)^2\}$.
 - (c) V : functions on \mathbb{R} , $S = \{1, e^{ax}, xe^{ax}\}$.
 - (d) V : functions on \mathbb{R} , $S = \{1, \cos 2x, \sin x\}$.
10. Consider the vector space \mathbb{R} over the field \mathbb{Q} . Calculate its dimension.
11. Give an example of a vector space V , a subspace S of V , and a basis B of V which does not contain a basis of S as a subset.