Homework 11

Analysis and Linear Algebra I (Autumn 2018)
Indian Institute of Science

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October 26, 2018

1. Suppose f is continuous and $g(x) = \displaystyle\int_0^x f(t)\,\mathrm{d}t$. Prove that $\displaystyle\int_0^x f(t)(x-t)\,\mathrm{d}t = \int_0^x g(t)\,\mathrm{d}t$.
2. Exercises 2, 5, 7, 11, 13, 19 of Section 5.8 in the textbook.
3. Exercises 3, 5, 11, 14 of Section 5.10 in the textbook.
4. Find all constants c such that $\log x = c + \displaystyle\int_e^x \frac{1}{t}\,\mathrm{d}t$ for all $x > 0$.
5. Suppose $f(x)$ is continuous for all $x > 0$ and has the property that $\displaystyle\int_x^{xy} f(t)\,\mathrm{d}t$ is independent of x for all $x, y > 0$. If $f(2) = 2$, compute $\int_1^x f(t)\,\mathrm{d}t$ for all $x > 0$.
6. Show that the absolute value of the remainder $E_{2n}(x)$ for the Taylor polynomial of $\sin x$ around $a = 0$ is $|E_{2n}(x)| \leq \dfrac{|x|^{2n+1}}{(2n+1)!}$.
7. Approximate $9^{1/3}$ by using a second order Taylor polynomial of an appropriate function around $a = 2$. Find a bound on the absolute error using **rational numbers only**.